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NOTE ON THE SIGNIFICANCE OF THE SIGNS + AND — BEFORE THE RADICAL $\sqrt{\ }$.

BY PROF. C. H. JUDSON, FURMAN UNIVERSITY, GREENVILL, S. C.

It is usual with Algebraists to write the roots of the quadratic $x^2 - 2px = q$ in this form; $x = p \pm \sqrt{(q + p^2)}$: whence it is inferred that the sign + before the radical indicates that the *positive root only* is to be taken, whilst the sign — indicates that the *negative root only* is to be taken. This is the express teaching of the ablest Algebraists. See Wood's Algebra by Lund, Cambridge, Eng., page 113—Todhunter's Algebra, page 170—Peacock's Symbolical Algebra, Vol. II, page 106—Hackley's Algebra, N. Y., page 214.

Wood and Todhunter give the following example:

$$x + \sqrt{(5x + 10)} = 8, \;.\;.\;.\;.\;.\;. \quad (1)$$

or $\sqrt{(5x + 10)} = 8 - x$; (2) whence $5x + 10 = 64 - 16x + x^2$ or $x^2 - 21x = -54$, whence $x = 18$ or 3. "But," they say, "it appears on trial that 18 does not satisfy the original equation, (1), but it belongs to the equation $x - \sqrt{(5x + 10)} = 8$." Is this doctrine defensible? Let us examine it.

Equation (2) shows that if $x = 3$ then $\sqrt{(5x + 10)} = 8 - 3$, or $+5$ and (1) becomes $3 + [5] = 8$, and the equation is *verified.*

If $x = 18$, then (2) gives $\sqrt{(5x + 10)} = 8 - 18 = -10$, and (1) is also verified, thus $18 + [-10] = 8$.

Hence we infer that the + sign before the radical indicates that the true root is to be taken with its proper sign; whilst the — sign indicates that the true root is to be taken with its sign changed.

If the original equation had been $x - \sqrt{(5x + 10)} = 8 \;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (1)$ or $x - 8 = \sqrt{(5x + 10)}$, (2) we should still have had $x^2 - 21x = -54$ and $x = 3$ or 18, as before. But now we have $\sqrt{(5x + 10)} = 3 - 8 = -5$ and (1) is verified; $3 - [-5] = 8$: also $\sqrt{(5x + 10)} = 18 - 8 = 10$, and (1) becomes $18 - [10] = 8$.

This last case shows conclusively that the — sign before the radical does *not* indicate that the negative root only is to be taken, but that the *true* root is to be taken with its sign *changed.*

If we take the equation $x + 5\sqrt{x} = -6$, we have $\sqrt{x} = -\frac{5}{2} \pm \frac{1}{2} = -3$, or -2, and $x = 9$ or 4. But the solution shows that the *negative* root is to be taken in both instances. If we take $x - 5\sqrt{x} = -6$ we have $\sqrt{x} = \frac{5}{2} \pm \frac{1}{2} = 3$ or 2 and $x = 9$ or 4; but now the *positive* roots only are to be taken, notwithstanding the — sign before the radical.

According to the received doctrine, the foregoing equations have absolutely no roots, real or imaginary. And this is maintained by the writers named. Thus also Young (Theory and Solution of Equations of Higher Orders, page 26,) says, "no expression, real or imaginary, can satisfy the equation $2x - 5 + \sqrt{(x^2 - 7)} = 0$." But why should the + sign before the radical indicate that the positive root is taken, any more than the + sign understood before the $2x$ indicates that the positive value only of x is to be taken? The foregoing examples and reasoning appear to be *conclusive*; and if the opposite doctrine—that generally received—can be maintained on any other ground than that of high authority, I should be glad to see it presented in the ANALYST.

A DEMONSTRATION OF THE BINOMIAL THEOREM FOR NEGATIVE EXPONENTS.

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Let $(1+x)^{-\frac{m}{n}} = A_0 + A_1x + A_2x^2 + A_3x^3 + \ldots\ldots\ldots\ldots \quad (1)$

$(1+x')^{-\frac{m}{n}} = A_0 + A_1x' + A_2x'^2 + A_3x'^3 + \ldots\ldots\ldots\ldots \quad (2)$

Put $u^{-m} = (1+x)^{-\frac{m}{n}}, \quad u_1^{-m} = (1+x')^{-\frac{m}{n}}$,

then $u^n = 1 + x, \quad u_1^n = 1 + x', \quad u^n - u_1^n = x - x'. \ldots\ldots\ldots \quad (3)$

Subtract (2) from (1), then

$$u^{-m} - u_1^{-m} = -\frac{u^m - u_1^m}{u^m u_1^m} = A_1(x - x') + A_2(x^2 - x'^2) + A_3(x^3 - x'^3). \, .$$

Divide the second member by $u^n - u'^n$, and the third member by $x - x'$, Eq. (3). Then

$$-\frac{u^m - u_1^m}{u^m u_1^m(u^n - u_1^n)} = -\frac{1}{u^m u_1}\left(\frac{u^{m-1} + u^{m-2}u_1 + u^{m-3}u_1^2 + \ldots + u_1^{m-1}}{u^{n-1} + u^{n-2}u_1 + u^{n-3}u_1^2 + \ldots + u_1^{n-1}}\right)$$
$$= A_1 + A_2(x + x') + A_3(x^2 + xx' + x'^2) + \ldots$$

Let $x' = x$, then $u = u'$, and

$$-\frac{mu^{m-1}}{nu^{n-1}u^{2m}} = -\frac{m}{n}\cdot\frac{1}{u^n u^m} = -\frac{m}{n}\left(1 + x\right)^{-\frac{m}{n}-1} = A_1 + 2A_2x + \, . \, .$$

Multiply both members by $(1 + x)$, and both members of (1) by $m \div n$, then, since $A_0 = 1$,

$$0 = \tfrac{m}{n}(1 + A_1x + A_2x^2 + \ldots + A_px^p + \ldots) + (1 + x)[A_1 + 2A_2x + 3A_3x^2 + \ldots + (p+1)A_{p+1}x^p + \ldots]$$